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## A SPECTRUM ANALYSIS OF THE LONG-SWING HYPOTHESIS\*

By E. PHILIP HOWREY<sup>1</sup>

### 1. INTRODUCTION

THE LONG-TERM ECONOMIC GROWTH of the American economy over the past century has frequently been characterized as expansion at a relatively constant rate. It is widely accepted that for some purposes this is a useful abstraction from the rather wide fluctuations observed in the annual growth rate. In more detailed studies of the growth of the American economy, a certain amount of interest has centered on the problem of isolating regular fluctuations about the long-term average value of the rate of expansion. Although some of the variation in the rate of growth is thought to be accounted for by the ordinary business cycle, several studies have suggested that the rate of growth accelerates and decelerates in a fairly regular pattern of some twenty years duration. These long swings are considered to be distinct from and independent of the shorter business cycle.

This paper applies the technique of spectrum analysis to the problem of determining the statistical significance of long swings in the rate of growth of output and other related macro-economic variables. In the next section alternative methods of time series analysis are compared and the effect of low-pass filtering operations is considered. The empirical results of the study are presented in Section 3, and the main conclusions are summarized in the final section.

### 2. TIME-SERIES ANALYSIS AND FILTERING OPERATIONS

The long-swing hypothesis is concerned with the existence of fluctuations of duration ranging between fifteen and twenty-five years.<sup>2</sup> The hypothesis has been formulated alternatively in terms of the level, rate of growth and deviation from trend of various economic variables. The usual method which is used to isolate long swings in a series is first to low-pass filter the series in order to attenuate the short-run fluctuations and then to mark off

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<sup>2</sup> Although Kuznets [16, (423)] suggests an average periodicity of twenty years for the long swing, there is less than universal agreement on the duration of these fluctuations. Abramovitz [3, (419)], on the basis of his study of U. S. data, suggests an average duration of fourteen years for the long swing. For a discussion of some of the difficulties involved in an attempt to specify precisely the duration of the long swing, see Hatanaka and Howrey [12].

the peaks and troughs in the *filtered* series. This chronology of peaks and troughs together with an estimate of the amplitude of the swings is used to determine whether the *original* series contains a long-swing component.

Apart from the subjectivity which is often involved in determining the peaks and troughs of the series, there are two points which should be considered in connection with this approach to the analysis of a time series. First, the determination of the "period" of a time series by counting peaks and troughs is but one of several alternative techniques which may be used for this purpose. Second, and more important, the inference of a period in the original series from estimates obtained from the filtered series may be misleading.

These two points are developed in this section with the following approach: First, the assumptions used throughout the paper are set out. Then four different measures of periodicity are introduced and illustrated with reference to a second-order autoregressive scheme. Finally, the effect of low-pass filtering the series before estimating the periodicity is considered. The major conclusion which emerges is that, in general, it is not valid to infer that the period of oscillation which is isolated in a filtered series is identical to the period of the original (unfiltered) series.

*Assumptions and notation.* Throughout this section it is assumed that the series being analyzed is a realization  $\{x_t; t = 1, 2, \dots, n\}$  of a stationary Gaussian process. The Gaussian assumption facilitates the computation of expected value of the "period" of the time series. In order to derive what is referred to below as the spectrum period of the process, the power spectrum is introduced:<sup>3</sup>

$$(1) \quad f(\omega) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \gamma(s) \cos \omega s \quad (-\pi \leq \omega \leq \pi),$$

where  $\gamma(s)$  denotes the autocovariance function of the process.

An example of a simple generating process which is used for illustrative purposes in this section is the second-order autoregressive process

$$(2) \quad x_t + ax_{t-1} + bx_{t-2} = \varepsilon_t \quad (a^2 < 4b < 4),$$

where  $\{\varepsilon_t\}$  is a sequence of normal random variables with mean zero and variance  $\sigma_\varepsilon^2$ . The inequalities guarantee that the characteristic roots of the difference equation are complex and less than unity in absolute value. The complete solution of (2) is given by<sup>4</sup>

$$(3) \quad x_t = D^t(A \cos \theta t + B \sin \theta t) + \sum_{j=0}^{\infty} \xi_j \varepsilon_{t-j+1},$$

where

$$\begin{aligned} D &= \sqrt{b}, \\ \theta &= \cos^{-1}(-a/2D), \\ \xi_j &= 2(4b - a^2)^{-\frac{1}{2}} D^j \sin j\theta, \end{aligned}$$

<sup>3</sup> A good introduction to spectral analysis is given in Granger and Hatanaka [11] and Jenkins [13].

<sup>4</sup> This is discussed fully in Kendall [14].

and  $A$  and  $B$  are constants determined by the initial conditions. The period of the solution of the homogeneous part of (2),  $2\pi/\theta$ , is referred to below as the autoregressive period of the scheme. Since  $b < 1$ , the first term in the solution approaches zero as  $t$  increases so that asymptotically

$$(4) \quad x_t = \sum_{j=0}^{\infty} \hat{\xi}_j \varepsilon_{t-j+1}.$$

It is the periodicity of the sequence  $\{x_t\}$  as determined by this last expression which is considered for illustrative purposes.

*Measures of periodicity.* Consider now the problem of determining the period of an observed time series  $\{x_t\}$ . At least four ways of measuring the period of a series are available. These include

- (a) the mean-distance between peaks (troughs),
- (b) the mean-distance between upcrosses (downcrosses),
- (c) the correlogram period and
- (d) the spectrum period.

In general, the expected values of (a)–(d) differ from each other. Hence, one's findings may depend critically on the way in which periodicity is measured. Since the empirical results of this study are presented in the form of estimates of the spectrum, it is of interest to compare the spectrum with the other three, more traditional, methods of characterizing a time series.<sup>5</sup>

(a) *The mean-distance between peaks.* The expected mean-distance between peaks, where a peak is said to occur at time  $t$  if  $x_{t-1} \leq x_t \geq x_{t+1}$ , can be determined in the following way. Let  $p$  denote the probability that  $x_t$  is a relative maximum (peak):

$$p = \Pr \{ \lambda_t \leq 0, \lambda_{t+1} \geq 0 \},$$

where

$$\begin{aligned} \lambda_t &= x_{t-1} - x_t, \\ \lambda_{t+1} &= x_t - x_{t+1}. \end{aligned}$$

Then in a series of  $N$  observations, one would expect to find  $Np = n$  peaks. The mean-distance between peaks is thus  $N/n = 1/p$ , i.e., the inverse of the probability that  $x_t$  is a peak. For a normal series  $p = \cos^{-1} \tau/2\pi$  where  $\tau$  is the correlation between  $\lambda_t$  and  $\lambda_{t+1}$ . In terms of the autocorrelation coefficients of the original series, the mean-distance between peaks is given by

$$(5) \quad P_1 = 2\pi/\cos^{-1} \left[ \frac{-1 + 2\rho(1) - \rho(2)}{2(1 - \rho(1))} \right],$$

where  $\rho(s) = \gamma(s)/\gamma(0)$  is the correlation between  $x_t$  and  $x_{t+s}$ . For the autoregressive process (2), the mean-distance between peaks is given by

<sup>5</sup> The following discussion of the mean-distance between peaks, mean-distance between upcrosses, and the correlogram period is based on Kendall [15] to which the reader is referred for a more detailed discussion and derivation.

$$(6) \quad P_1 = 2\pi/\cos^{-1} \left[ \frac{b^2 - (1+a)^2}{2(1+a+b)} \right].$$

(b) *The mean-distance between upcrosses.* An upcross is said to have taken place between  $t-1$  and  $t$  provided  $x_{t-1} \leq \mu \leq x_t$ , where  $\mu$  is the mean of the series. By an argument analogous to that given for the mean-distance between peaks, the expected mean-distance between upcrosses is

$$(7) \quad P_2 = 2\pi/\cos^{-1} \rho(1).$$

For the autoregressive process this may be written as

$$(8) \quad P_2 = 2\pi/\cos^{-1} \left[ \frac{-a}{1+b} \right].$$

It is of interest to note that the mean-distance between upcrosses depends only on the first-order autocorrelation of the series, whereas mean-distance between peaks depends on the first- and second-order autocorrelation coefficients of the series. Both of these measures are based on the probability of occurrence of a given event (peak or upcross) so that the *stochastic* nature of the series is explicitly taken into consideration. However, in these definitions no subsidiary constraints such as conditions which have the effect of reducing "ripple" have been imposed.<sup>6</sup> Although this limits to some extent the applicability of these two measures to economic time series, they are suggestive and lead to interesting comparisons with the correlogram and spectrum periods.

(c) *The correlogram period.* A third measure of the periodicity of a series can be derived from the correlogram. In general, the correlogram period is defined as the mean-distance between troughs (peaks) or downcrosses (upcrosses) in the sequence of serial correlation coefficients. For the autoregressive process (2), the theoretical values of the serial correlation coefficients are given by

$$(9) \quad \rho(s) + a\rho(s-1) + b\rho(s-2) = 0 \quad (s \geq 1);$$

that is, the serial correlation coefficients are generated by the homogeneous part of the difference equation. It follows that the correlogram oscillates with a periodicity which is identical to the autoregressive period, namely,

$$(10) \quad P_3 = 2\pi/\cos^{-1} \left[ \frac{-a}{2\sqrt{b}} \right].$$

The fact that the correlogram period is equal to the period of the solution of the homogeneous part of the difference equation probably accounts for the intuitive appeal of this measure.

(d) *The spectrum period.* The spectrum period is defined as the inverse of the frequency at which the power spectrum exhibits a relative peak (provided one exists). The power spectrum of the autoregressive process

<sup>6</sup> An additional constraint on the mean-distance between peaks which has the effect of reducing ripple, namely,  $x_t \geq x_{t+h}$ , has been discussed by Dodd [8].

(2) is given by<sup>7</sup>

$$(11) \quad f_x(\omega) = |1 + ae^{-i\omega} + be^{-i2\omega}|^{-2} f_\varepsilon(\omega) \quad (0 \leq \omega \leq \pi),$$

or, equivalently, by

$$(11') \quad f_x(\omega) = \frac{\sigma_\varepsilon^2}{2\pi(1 + a^2 + b^2 + 2a(1 + b)\cos\omega + 2b\cos 2\omega)}.$$

The theoretical spectrum of the autoregressive with  $a = -1.1$ ,  $b = 0.5$  and  $\sigma_\varepsilon^2 = \pi$  is shown in Figure 1. This power spectrum exhibits a relative peak at  $\omega = \cos^{-1}[-a(1 + b)/4b]$ , so that the spectrum period of the autoregressive is<sup>8</sup>

$$(12) \quad P_1 = 2\pi/\cos^{-1}\left[\frac{-a(1 + b)}{4b}\right].$$

The interpretation of the spectrum period is relatively straightforward. It is simply the inverse of the center frequency of that band of frequencies which makes the largest contribution to the variance of the series.

In general, each of these measures of periodicity is different. The extent to which these measures diverge from one another depends on the exact nature of the generating process. With reference to the autoregressive process, all four of these measures yield the same result if  $b = 1$ , in which case the process contains a deterministic component since the first term in the general solution (3) does not damp out. With both  $a$  and  $b$  equal to zero,  $\{x^t\}$  is simply a random series. The mean-distance between peaks is

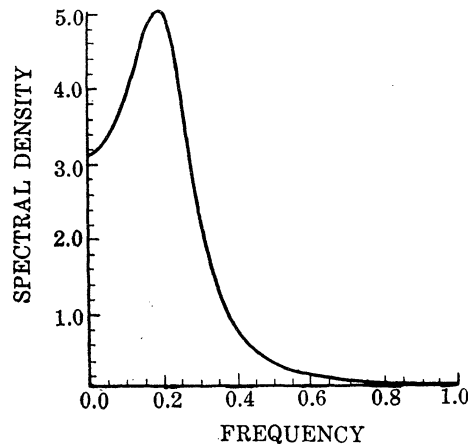


FIGURE 1  
THEORETICAL SPECTRUM OF  
 $x(t) - 1.1x(t - 1) + 0.5x(t - 2) = \varepsilon(t)$

<sup>7</sup> The method by which this expression is obtained is described in Granger and Hatanaka [11, (35-7)].

<sup>8</sup> The expression for  $\omega$  is obtained by setting the derivative of  $f_x(\omega)$  given in (11) equal to zero and solving for  $\omega$ .

three units of time and the mean-distance between upcrosses is four units of time, both of which are well-known results. The correlogram does not oscillate in this case since  $\rho(s) = 0, (s > 0)$ , and the power spectrum is flat so that neither the correlogram period nor the spectrum period is defined in this case. The values of the period as measured by each of these methods are set out in Table 1 for different values of  $a$  and  $b$  of the autoregressive process. From this table it is obvious that the period which is contained in a time series depends critically on the way in which periodicity is defined and measured.

TABLE 1  
THEORETICAL PERIODICITY OF SERIES GENERATED BY  
THE AUTOREGRESSIVE PROCESS (2)

$a$	-1.5	-1.7	-1.615	-1.52	-1.1
$b$	0.9	0.8	0.7	0.6	0.5
$P_1$	7.90	8.69	7.36	6.43	4.96
$P_2$	9.51	18.76	19.79	19.79	8.40
$P_3$	9.53	19.85	23.73	32.31	9.25
$P_4$	9.56	21.16	31.79	$\infty$	10.46

A tacit assumption underlying this discussion of measures of periodicity is that the series under consideration consists of a sequence of fluctuations which can be characterized by a single average periodicity. This does not mean that each successive fluctuation must be exactly the same length, but that the dispersion about the average is not so great that the average is meaningless. Provided this assumption is satisfied, the empirical implementation of each of the measures of periodicity is straightforward. With respect to the mean-distance between peaks (upcrosses), an obvious procedure is to mark off the peaks (upcrosses) and then determine the average distance between them. For the correlogram and spectrum, the theoretical serial correlation coefficients could be replaced by their estimates.<sup>9</sup>

An economic time series, however, is not likely to satisfy such an assumption for it implies that the series contains a single period. It is more reasonable to assume that economic time series consist of superimposed fluctuations, each of which can be characterized by an average periodicity. Indeed, the long-swing hypothesis is specifically concerned with the existence of a fluctuation which is longer in duration than and superimposed upon the ordinary business cycle. With a series of superimposed variations, the problem of decomposing the series into meaningful components immediately arises. If, for example, periodicity is measured by the mean-distance between peaks, it is necessary to establish a criterion that will enable the investigator to distinguish between business-cycle peaks and long-swing peaks in the time series under consideration. One method that has been used in this connection, low-pass filtering, will now be considered.

<sup>9</sup> This is not the usual procedure which is used to estimate the spectrum.

*Filtering operations.* In most earlier studies of the long-swing hypothesis, an indirect method has been used to distinguish between long swings and business cycles in the series of observations. The usual procedure involves applying a low-pass filter to the original series in order to reduce or eliminate short-run fluctuations.<sup>10</sup> The filtered series is then analyzed as if it were identical to the original series but with the high-frequency components removed. A low-pass filter which has often been used in this connection is the simple moving average of length  $2m + 1$

$$(13) \quad y_t = \sum_{k=-m}^m x_{t+k} / (2m + 1).$$

In this expression  $\{y_t\}$  denotes the filtered series, and  $\{x_t\}$  denotes the original series. The effect of the filtering operation described by (13) on each of the four measures of periodicity enumerated above will now be examined and illustrated with reference to the autoregressive process (2).

Each of the measures of periodicity depends on the serial correlation coefficients of the series which is being analyzed. The general form of the autocovariance function of the filtered series obtained from an original series by (13) is

$$(14) \quad \gamma^y(s) = \sum_{k=-m}^m (2m + 1 - |k|) \gamma^x(k + s),$$

where  $\gamma^y(s)$  and  $\gamma^x(s)$  denote, respectively, the autocovariance function of the filtered and original series. Given the autocovariance function of the original series, the filtered autocovariance and autocorrelation functions can be obtained. The effect of the filtering operation on the periodicity of the series is not, however, immediately apparent. A general argument in terms of the spectrum is given below, but for the other three measures of periodicity the effect of the filtering is merely suggested by referring to numerical examples. For expository purposes it is assumed that the original series is generated by the autoregressive scheme (2) and the filtered series is obtained from (13). The period of the original and filtered series for various values of the autoregressive coefficients  $a$  and  $b$  and different values of  $m$ , where  $m$  determines the length of the filter, are set out in Table 2.<sup>11</sup>

This table suggests that the mean-distance between peaks,  $P_1$ , and the mean-distance between upcrosses,  $P_2$ , are both increased by the simple moving average filter. For example, with  $a = -1.7$  and  $b = 0.8$ , the theoretical mean-distance between peaks in the original series is 8.7, and the mean-distance between upcrosses is 18.8 units of time. The expected mean-distance between peaks in the five-item moving average of this series ( $m = 2$ ) is 15.3, while the mean-distance between upcrosses is 22.3 units of time.

<sup>10</sup> For example, Kuznets [16] has used a low-order moving average filter to eliminate short-run fluctuations from the series. Various studies of long-swing hypothesis in which some filtering technique has been used are enumerated by Adelman [5].

<sup>11</sup> The values for  $a$  and  $b$  in this table are the same as those used in Table 1. When  $m = 0$ , the length of the filter is  $2m + 1 = 1$  so that the filtered and original series are identical.



TABLE 2  
PERIOD OF ORIGINAL AND FILTERED AUTOREGRESSIVE SERIES  
FOR DIFFERENT VALUES OF  $a, b$  AND  $m$

$P_j$	$m$	$a$	-1.5	-1.7	-1.615	-1.52	-1.1
		$b$	0.9	0.8	0.7	0.6	0.5
$P_1$	0		7.9	8.7	7.4	6.4	5.0
	2		9.6	15.3	14.6	13.6	8.6
	3		10.0	17.0	16.6	15.8	8.8
	4		9.9	18.3	18.2	17.4	8.4
$P_2$	0		9.5	18.8	19.6	19.8	8.4
	2		10.7	22.3	25.6	27.8	14.1
	3		12.4	24.2	28.5	31.6	17.7
	4		16.7	26.5	31.7	35.5	20.5
$P_3$			9.5	19.9	23.7	32.3	9.3
$P_4$	0		9.6	21.2	31.8	$\infty$	10.5
	2		9.6	22.2	36.4	$\infty$	26.7
	3		9.8	22.2	44.4	$\infty$	$\infty$
	4		$\infty$	23.6	80.0	$\infty$	$\infty$

For higher-order moving averages ( $m = 3, 4$ ) the period of the filtered series is somewhat longer in this example. It is apparent from this table that the period of the filtered series is greater than the period of the original series, at least for a certain range of values of  $a, b$  and  $m$ . This indicates that in general *it is not valid to conclude that the period which is determined by marking off peaks or upcrosses in the filtered series is identical to the period of the original series.*

The correlogram period is not subject to this difficulty in the case of the autoregressive process. The reason for this is that the autocorrelation coefficients of the original series are generated by (7) which has as its solution<sup>12</sup>

$$(15) \quad \rho(k) = D^k \frac{\sin(k\theta + \Psi)}{\sin \Psi},$$

where

$$\begin{aligned} D &= \sqrt{b}, \\ \theta &= \cos^{-1}(-a/2D), \\ \tan \Psi &= \frac{1+b}{1-b} \tan \theta. \end{aligned}$$

According to (14) the autocorrelation coefficients of the filtered series are linear combinations of the autocorrelation coefficients of the original series. Since the autocorrelation coefficients of the original series all have the same

<sup>12</sup> Cf. Kendall [15, (26)].

period and the same damping factor, it follows that the period of oscillation of the correlogram of the filtered series is the same as that of the original series.<sup>13</sup> For this reason only one number is given in Table 2 for the correlogram period.

The spectrum period exhibits the same tendency as the mean-distance between peaks when a moving-average filter is applied to a series. In this case a general argument is much simpler to construct than in the preceding cases. The power spectrum of the filtered series,  $f_y(\omega)$ , is related to the power spectrum of the original series by

$$(16) \quad f_y(\omega) = |G(\omega)|^2 f_x(\omega) \quad (-\pi \leq \omega \leq \pi),$$

where  $G(\omega)$ , the gain of the  $(2m + 1)$ -item moving average, is

$$(17) \quad G(\omega) = \sum_{s=-m}^m e^{i\omega s} / (2m + 1).$$

The gain of a five-item moving average is shown in Figure 2. It is apparent that the low frequencies are passed by the filter and the high frequencies are rejected.

Suppose that the spectrum of the original series,  $f_x(\omega)$ , exhibits a peak at  $\omega$ , i.e.,

$$(18) \quad \left. \frac{df_x(\omega)}{d\omega} \right|_{\hat{\omega}} = 0.$$

The corresponding peak in the filtered series occurs at  $\bar{\omega}$ , where  $\bar{\omega}$  is determined by solving

$$(19) \quad \frac{df_y(\omega)}{d\omega} = f_x(\omega) \frac{d|G(\omega)|^2}{d\omega} + |G(\omega)|^2 \frac{df_x(\omega)}{d\omega} = 0$$

or, equivalently, by solving

$$(19') \quad \frac{df_x(\omega)}{d\omega} = -f_x(\omega) \frac{d \ln |G(\omega)|^2}{d\omega}.$$

Since  $|G(\omega)|^2$  is a positive but decreasing function of  $\omega$  for  $\omega < 2\pi/5$ , it follows that  $d \ln |G(\omega)|^2 / d\omega < 0$  for  $0 \leq \omega < 2\pi/5$ . Therefore the right-hand side of (19') is positive for  $\omega < \hat{\omega}$  and negative for  $\omega > \hat{\omega}$ . A comparison of (18) and (19') indicates that  $\bar{\omega} < \hat{\omega}$  provided  $\hat{\omega} < 2\pi/5$ . If the original series contains an important component of periodicity greater than five units of time per cycle, the filtered series will contain an important component of duration longer than that of the original series.<sup>14</sup> This point is illustrated in Figure 3 which shows the theoretical spectrum of a five-item moving average of the series generated by the autoregressive process with  $a = -1.1$  and  $b = 0.5$ .

<sup>13</sup> See, for example, Allen [6, (129-31)].

<sup>14</sup> In general, a  $(2m + 1)$ -item moving average "shifts" peaks in the spectrum which are located below  $1/(2m + 1)$  cycle per period to still lower frequencies. It should be noted that this has nothing to do with aliasing which is described by Blackman and Tukey [7, (31-3)] and which is explored by Taubman [21] in connection with the long-swing hypothesis.

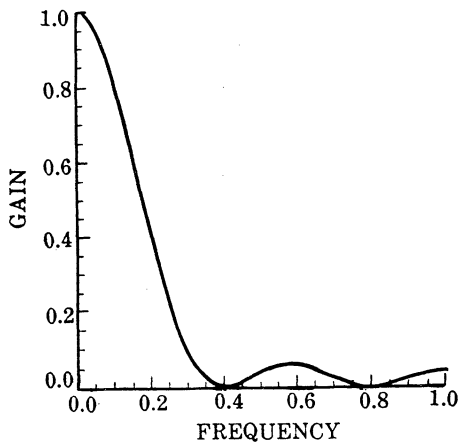


FIGURE 2

GAIN OF AN UNWEIGHTED FIVE  
ITEM MOVING AVERAGE

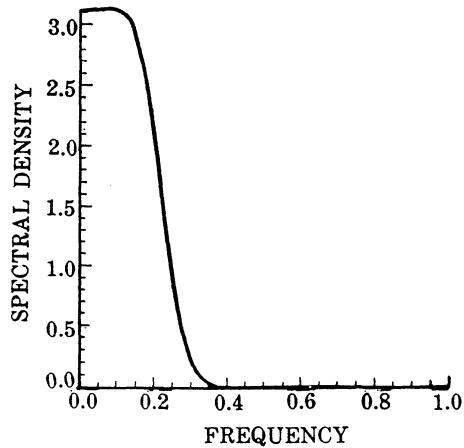


FIGURE 3

THEORETICAL SPECTRUM OF A FIVE  
ITEM MOVING AVERAGE OF  
 $x(t) - 1.1x(t-1) + 0.5x(t-2) = \epsilon(t)$

*Summary.* This discussion of the three measures of periodicity which are not invariant with respect to filtering operations, namely, the mean-distance between peaks, the mean-distance between upcrosses and the spectrum period, suggests an important point in connection with the long-swing hypothesis. Specifically, a major cycle with a periodicity of between eight and eleven years in the original series may appear as a long swing with a periodicity of between fifteen and twenty-five years in the filtered series. This point is strikingly illustrated by the numerical results set out in Table 2. Since several earlier studies of the long-swing hypothesis have used filters that are identical or similar to those described above, the results of these studies must be interpreted with extreme caution. In order to avoid the possibility of drawing misleading inferences from estimates obtained from a filtered series, it is necessary to consider the effect of the filter.

One of the distinct advantages of the spectrum-analytic approach to time-series analysis is that this sort of adjustment problem can easily be handled. In those cases in which a filter is used, the estimates of the spectrum can be adjusted for the effect of the filter in a relatively straightforward way.<sup>15</sup> In many cases, however, it is not necessary to process the series in order to eliminate the short cycles before proceeding with the estimation. In such cases the relative importance of the long swing can be compared directly with that of the major and minor business cycle.

### 3. SPECTRUM ESTIMATION AND THE LONG-SWING HYPOTHESIS

In this section the empirical results of this study presented in the form of

<sup>15</sup> For a discussion of situations in which it might be advisable to filter the series before proceeding with the estimation, see Blackman and Tukey [7, (39-43)].

estimates of the spectral density functions of a number of macro-economic time series are described. The particular form of the long-swing hypothesis with which this study is concerned is the growth-rate variant. This choice was made, in part, in view of the stationarity assumption of spectrum analysis. While the absolute level of most economic variables cannot possibly be considered to be generated by a stationary stochastic process because of the dominant trend in mean, the sequence of growth rates is somewhat less questionable.<sup>16</sup>

Each of the series analyzed was first transformed by computing relative rates of growth according to

$$(20) \quad y(t) = [x(t+1) - x(t)]/x(t),$$

where  $\{x(t)\}$  denotes the original series and  $\{y(t)\}$  denotes the series of growth rates. The spectrum densities of the growth-rate series were then estimated using a Parzen window.<sup>17</sup> During the course of the investigation, spectrum densities were estimated using  $T = 10, 20, 30, 40, 60$  and  $80$  lags. As the number of lags used in the estimation of the spectrum is increased, a sharper resolution of the frequency axis is possible, but this is achieved at the expense of an increase in the variance of the estimate. It was found that the twenty-lag estimates described reasonably well the main features of the spectra of the series dealt with here.

A number of previous investigators have suggested an average duration of fifteen to twenty-five years for the long swing, with an average duration of twenty years.<sup>18</sup> In the following diagrams the frequency band which corresponds to a fluctuation with an average periodicity of twenty years is centered on the tenth frequency point (i.e.,  $10\pi/100$  radians per year on  $1/20$  cycle per year). The long-swing hypothesis can be interpreted as stating that the variance-contribution of this band of frequencies is significantly greater than that of neighboring frequency bands. This intuitive statement of the hypothesis suggests that its rejection be based on the absence of a local peak in the spectrum near this long-swing frequency. For the use of the estimated spectrum as a descriptive statistic, this statement of the hypothesis seems to be adequate. However, a more precise formulation of the hypothesis in terms of conventional tests of significance is possible. The  $100(1 - 2\alpha)$  per cent confidence band for normally distributed independent random variables, referred to as white noise, can be determined from

$$\Pr \left\{ \chi_{1-\alpha}^2(l) \leq \frac{l\hat{f}(\omega)}{f(\omega)} \leq \chi_{\alpha}^2(l) \right\} = 1 - 2\alpha,$$

<sup>16</sup> Adelman [5] has experimented with residuals from a log-linear trend. Although this transformation might be expected to eliminate the trend in the mean, the trace of the residual series suggests that the variance of the residuals is not stationary. The non-stationarity of the growth rate series is in most instances less conspicuous, although perhaps no less real.

<sup>17</sup> The estimation procedure which was used here is that described by Parzen [20].

<sup>18</sup> See, for example, Kuznets [16] and Lewis and O'Leary [19].

where  $l$ , the equivalent degrees of freedom of each estimate, is determined by dividing the number of observations used in the estimation of the spectrum by  $T/4$  (i.e.,  $l = 4n/T$ ). These confidence limits provide a method for testing the hypothesis that the underlying process is random. Specifically, an estimate which lies outside the  $(100 - 2\alpha)$  per cent confidence limits is said to be significantly different from white noise at that level.

The general features of the spectral density functions estimated with  $T = 20, 30$  and  $40$  are set out in Table 3. For each truncation point used in the estimation, the location of the relative peaks in terms of years per cycle (ypc) and the estimated peak value are given. For expository purposes, the relative peaks are grouped in the table according to their location:  $15-\infty$  years per cycle, 9-15 ypc, 5-9 ypc, 3-5 ypc, and 2-3 ypc. The first number in each column indicates the center of the band in which the spectrum exhibits a relative peak. Where no number is shown, the spectrum does not exhibit a relative peak in the band. The peak value of the spectrum is shown directly below the location figure. A single (double) underscore denotes peaks which are significantly different from the spectrum of white noise at the 90 per cent (95 per cent) confidence level.

For example, the spectrum of Gross National Product series estimated with twenty lags exhibits local peaks in the bands centered on 11.8, 5.6 and 3.4 years per cycle. These fluctuations have relative amplitudes of 0.48, 0.78 and 0.77 respectively, none of which lies outside the upper 90 per cent white noise confidence limit. This summary presentation facilitates a comparison of spectrum estimates which differ from one another in the focussing power of the window used in their estimation.

The results of the estimation are shown in graphical form in Figures 4-17 for the truncation point  $T = 20$ . As suggested above, the twenty-lag estimates adequately reflect the general features of the thirty- and forty-lag estimates, so only the twenty-lag estimates are shown in graphical form. In addition to the estimated spectrum, the trace of the relative rate of growth of each series is also shown. These are included as a basis for making some judgment about the stationarity of the series from which the spectrum is estimated.<sup>19</sup>

<sup>19</sup> It will be noted that in the growth-rate series there are several extreme values. These are particularly evident in gross capital formation and gross nonfarm residential construction, for example. In order to determine whether the shape of the estimated spectrum is largely the result of these extreme values, two types of outlier adjustment procedures were employed. In both procedures an outlier is defined as an observation which lies more than  $k$  standard deviations from the mean of the series. In the first procedure the outlier is adjusted to lie exactly  $k$  standard deviations from the mean. In the second procedure the outlier is replaced by a linear interpolate between the two adjacent "inliers." The power spectra of these adjusted series were then estimated. With  $k = 2$ , the estimates of the spectrum density retained their original shape, except in the case of Inventory Investment which became almost flat. These outlier adjustment experiments, while not conclusive, do suggest that the estimates are not entirely due to the existence of extreme values in the series.

TABLE 3  
GENERAL FEATURES OF THE SPECTRUM ESTIMATES\*

Series (Variance)	Number of lags		Location and value of spectrum peaks				
			15-∞ ypc	9-15 ypc	5-9 ypc	3-5 ypc	2-3 ypc
1 Gross national product (0.0047)	20	duration amplitude		11.8 0.48	5.6 0.78	3.4 0.77	
	30	duration amplitude		11.1 0.52	5.6 0.90	3.4 0.94	2.4 0.41
	40	duration amplitude	20.0 0.47	10.5 0.56	5.7 0.97	3.4 1.08	2.5 0.46
2 Net national product (0.0069)	20	duration amplitude		11.1 0.42	5.6 0.73	3.3 0.87	2.4 0.44
	30	duration amplitude		10.5 0.46	5.6 0.87	3.3 1.03	2.5 0.53
	40	duration amplitude	18.2 0.38	10.0 0.51	5.7 0.97	3.4 1.17	2.5 0.62
3 Industrial production (0.0142)	20	duration amplitude			7.1 0.82	3.4 0.83	
	30	duration amplitude		12.5 0.59	6.7 0.94	3.4 1.05	2.8 0.66
	40	duration amplitude		13.3 0.63	6.5 1.04	3.4 1.26	2.8 0.74
4 Pig iron production (0.0574)	20	duration amplitude			6.2 0.60	3.4 0.96	2.8 0.69
	30	duration amplitude		13.3 0.44	6.2 0.72	3.4 1.26	2.7 0.83
	40	duration amplitude		14.3 0.48	6.2 0.82	3.4 1.56	2.7 0.94
5 Flow of goods to consumers (0.0025)	20	duration amplitude	25.0 0.60		5.7 0.54	3.6 0.52	2.5 0.62
	30	duration amplitude	22.2 0.68	9.5 0.43	5.7 0.60	3.6 0.62	2.5 0.80
	40	duration amplitude	22.2 0.77	9.5 0.46	5.9 0.65	3.4 0.67	2.5 0.93
6 Consumers' durables (0.0260)	20	duration amplitude		10.5 0.72	5.6 0.81	3.3 0.84	2.3 0.36
	30	duration amplitude		10.5 0.90	5.6 0.94	3.3 1.05	2.4 0.40
	40	duration amplitude	28.6 0.26	11.1 1.05	5.7 1.02	3.4 1.12	2.4 0.43

\* The sources of the various series are given at the end of the table. The value in parentheses following the description of the series is an estimate of the variance. Amplitudes with a single (double) underscore are peaks in the spectrum which are significantly different from white noise at the 90 per cent (95 per cent) confidence level.

Source: The data used in this study, with the exception of the Index of Industrial Production and Pig Iron Production, are the annual estimates underlying the series published in Kuznets [16]. Where more than one estimate was available, Kuznets' Variant III was used. The series are expressed in constant (1929) dollars

(Continued on next page)

TABLE 3  
(Continued)

Series (Variance)	Number of lags		Location and value of spectrum peaks				
			15-∞ ypc	9-15 ypc	5-9 ypc	3-5 ypc	2-3 ope
7 Consumers' semi- durables (0.0048)	20	duration amplitude	16.7 0.55		5.1 0.56	3.2 0.53	2.4 0.65
	30	duration amplitude	16.7 0.66		5.0 0.65	3.3 0.62	2.5 0.75
	40	duration amplitude	16.7 0.74	6.9 0.42	5.0 0.72	3.3 0.70	2.5 0.82
8 Gross capital formation (0.0574)	20	duration amplitude			5.7 <u>1.04</u>	3.2 0.63	2.2 0.38
	30	duration amplitude		11.1 0.69	5.4 <u>1.23</u>	3.2 0.71	2.2 0.41
	40	duration amplitude		11.1 0.73	5.6 <u>1.33</u>	3.1 0.77	2.2 0.42
9 Gross producers' durables (0.0412)	20	duration amplitude			7.1 <u>1.13</u>	3.4 <u>0.87</u>	2.1 0.18
	30	duration amplitude	22.2 0.31		7.7 <u>1.24</u>	3.4 <u>1.16</u>	2.2 0.18
	40	duration amplitude	22.2 0.36		8.0 <u>1.32</u>	3.4 <u>1.42</u>	2.2 0.19
10 Gross nonfarm residential construction (0.1440)	20	duration amplitude		11.8 <u>0.98</u>		3.4 0.43	2.3 0.46
	30	duration amplitude		11.8 <u>1.14</u>	5.6 0.61	3.5 0.46	2.4 0.48
	40	duration amplitude		12.5 <u>1.25</u>	5.3 0.63	3.5 0.51	2.4 0.54
11 Inventory investment (6.068)	20	duration amplitude	20.0 0.65			4.1 <u>0.89</u>	2.3 0.66
	30	duration amplitude	20.0 0.78	10.0 0.47		4.2 <u>1.07</u>	2.3 0.70
	40	duration amplitude	20.0 0.92	9.5 0.53		4.2 <u>1.25</u>	2.4 0.76
12 GNP per worker (0.0051)	20	duration amplitude		10.5 0.55	5.4 <u>0.85</u>	3.4 0.70	
	30	duration amplitude		10.5 0.52	5.6 <u>0.91</u>	3.4 <u>0.96</u>	2.4 0.42
	40	duration amplitude	20.0 0.50	10.5 0.56	5.7 <u>0.98</u>	3.4 <u>1.10</u>	2.5 0.47

and cover the period 1869-1955. The Index of Industrial Production with 1929 as the base year was constructed by splicing three series: Frickey's Index of Production for Manufacture [10, (54)] covering the period 1860-1914; Fabricant's Index of Output of Manufacturing Industries [9, (44)] covering the period 1915-1918; and the Federal Reserve Board's Index of Industrial Production [22 and 23] covering the period 1919-1960. The Pig Iron Production series was taken from [24] for the years 1860-1945 and from [25] for the years 1946-1961. Permission by S. Kuznets and the National Bureau of Economic Research to use these series is gratefully acknowledged.

TABLE 3  
(Continued)

Series (Variance)	Number of lags	Location and value of spectrum peaks					
		15-∞ ypc	9-15 ypc	5-9 ypc	3-5 ypc	2-3 ypc	
13 GNP per capita (0.0046)	20	duration amplitude		10.5 0.55	5.4 <u>0.85</u>	3.4 0.70	
	30	duration amplitude		10.5 0.67	5.6 <u>1.01</u>	3.4 0.85	2.4 0.43
	40	duration amplitude	22.2 0.34	10.5 0.76	5.7 <u>1.09</u>	3.4 <u>0.97</u>	2.4 0.40
14 Population (0.00003)	20	duration amplitude	∞ <u>4.60</u>			3.5 0.10	2.3 0.03
	30	duration amplitude	∞ <u>5.96</u>	10.5 0.73	4.9 0.08	3.5 0.12	2.3 0.03
	40	duration amplitude	∞ <u>6.87</u>	10.0 0.82	4.9 0.09	3.5 0.13	2.3 0.03

A comparison of the spectra which relate to national income and production with the spectra of the consumption and investment components of national product reveals several interesting points. The spectra of the income and production series (1-4 in Table 3) shown in Figures 4-7 exhibit major peaks in the ranges [5.6-7.1] and [3.3-3.4] years per cycle (ypc).<sup>20</sup> The latter peak which corresponds to the well known forty-month cycle is the more prominent of the two, at least in the Net National Product and Pig Iron Production series. Relatively weak peaks emerge in the intervals [10.0-14.3] and [2.4-2.8] ypc. Only when the truncation point is increased to 40 does anything remotely resembling a long swing emerge in these series; and then only in the GNP and NNP series. The long swing seems to be entirely absent from the two production series.

The estimates derived from the series relating to aggregate consumption (5-7 in Table 3), shown in Figures 8-10, are interesting in several respects. The spectra of the total consumption and consumers' semi-durables series are very weak (i.e., not statistically different from the spectrum of white noise) but do exhibit a relative peak in the [16.7-25.0] ypc range. The spectrum of the consumers' durables series is much like that of the income series in that relatively strong peaks emerge in the ranges [5.6-5.7] and [3.3-3.4] ypc. The major difference is that the major cycle of periodicity [10.5-11.1] ypc is much more pronounced in the consumers' durables series than in the income series. All this agrees reasonably well with the accepted notions about the volatility of the various components of consumption expenditure. The curious thing about these series is the emergence of a weak

<sup>20</sup> The notation [ $p_1$ - $p_2$ ] is used to denote the location of the relative peaks in the set of series under consideration. The value  $p_1$  is the shortest duration at which the spectrum exhibits a relative peak over all the estimates of all the series, and  $p_2$  is the longest duration over all the estimates of all the series in the group. The grouping of the relative peaks in this way is to a certain extent arbitrary, but the overall picture is relatively clear.



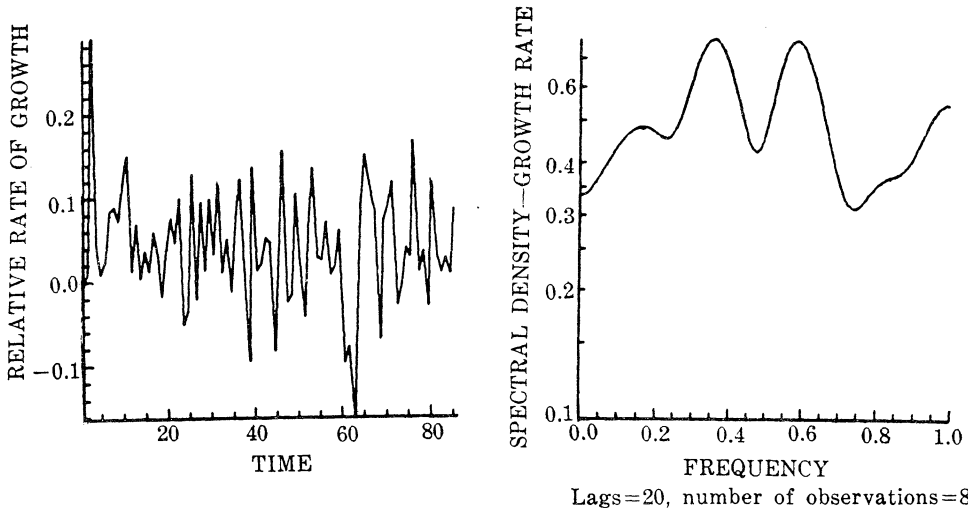


FIGURE 4

GROSS NATIONAL PRODUCT, 1869-1955

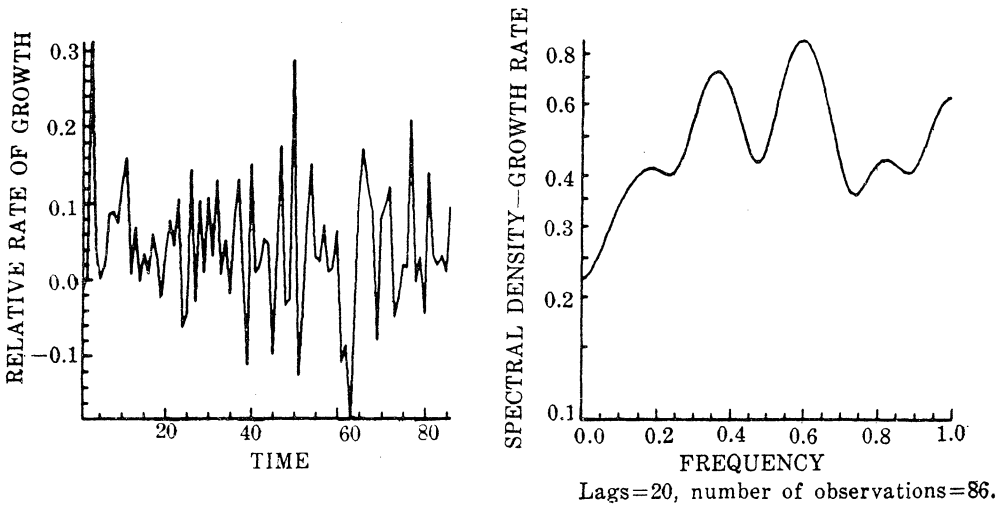


FIGURE 5

NET NATIONAL PRODUCT, 1869-1955

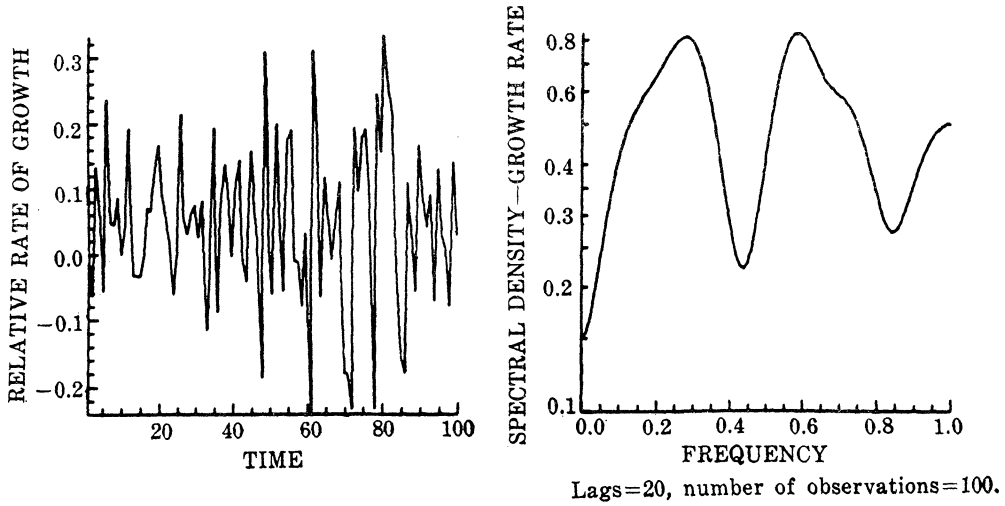


FIGURE 6  
INDEX OF INDUSTRIAL PRODUCTION, 1860-1960

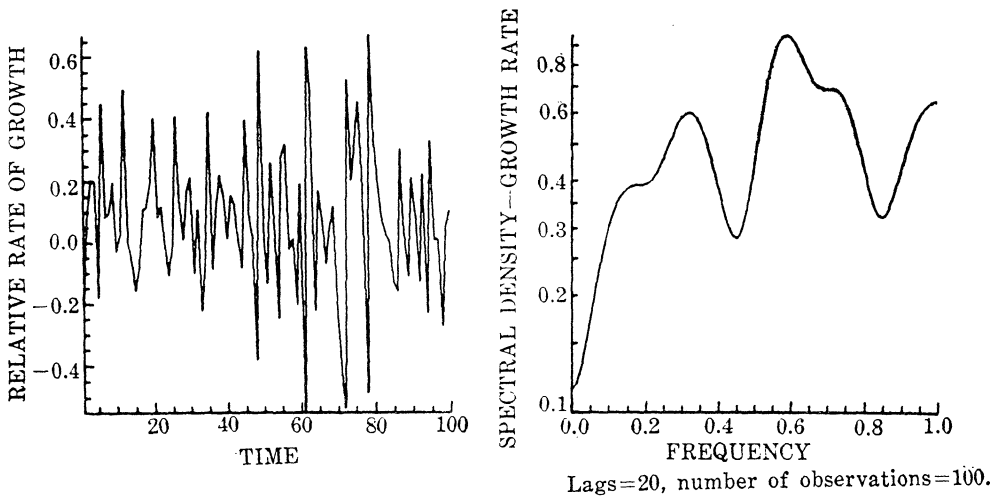


FIGURE 7  
PIG IRON PRODUCTION, 1860-1961

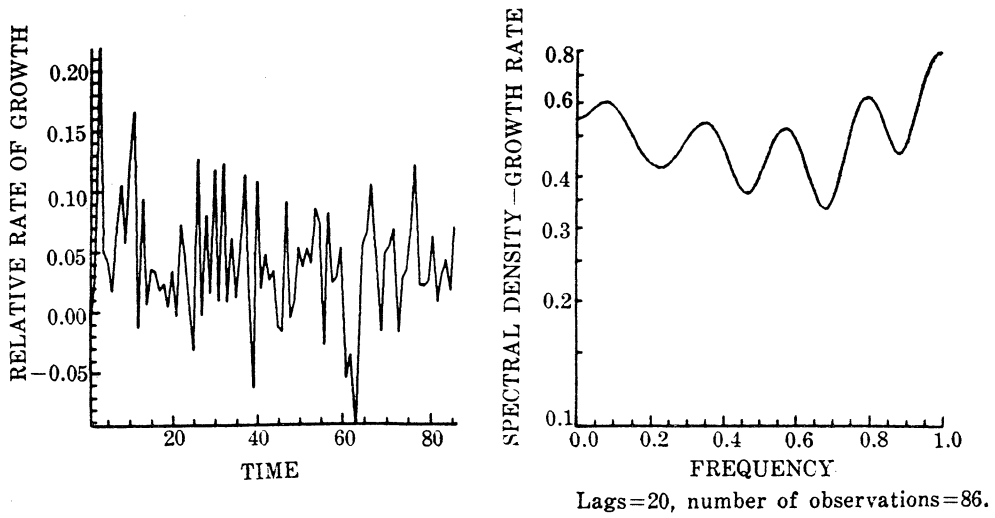


FIGURE 8

FLOW OF GOODS TO CONSUMERS, 1869-1955

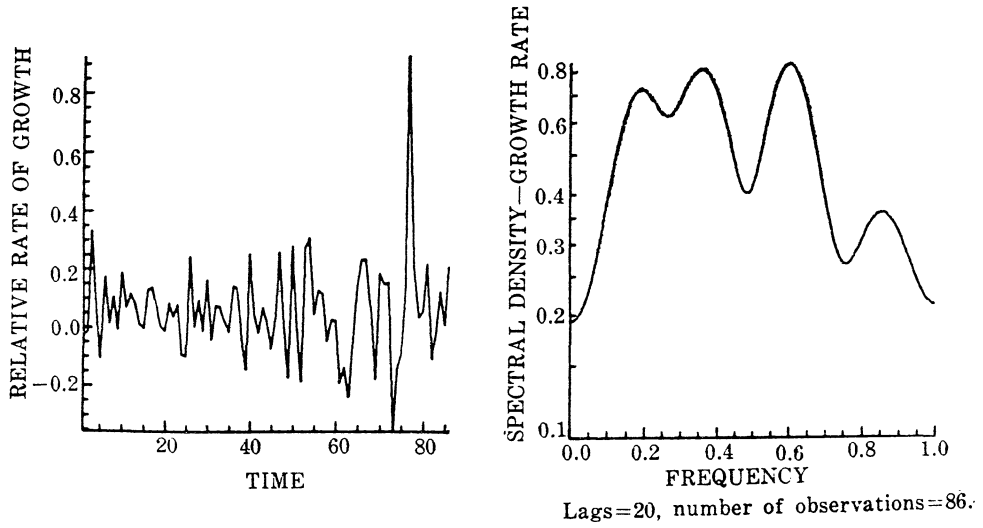


FIGURE 9

CONSUMERS' DURABLES, 1869-1955

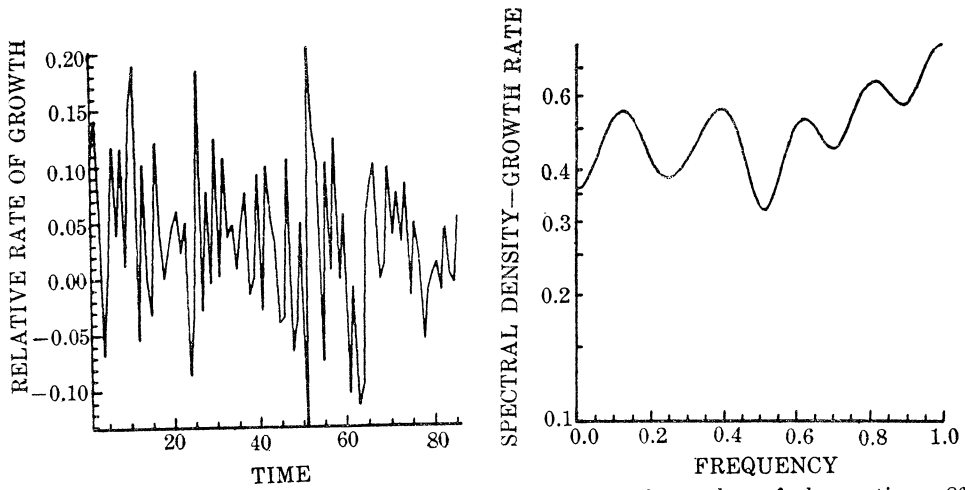


FIGURE 10

CONSUMERS' SEMI-DURABLES, 1869-1955

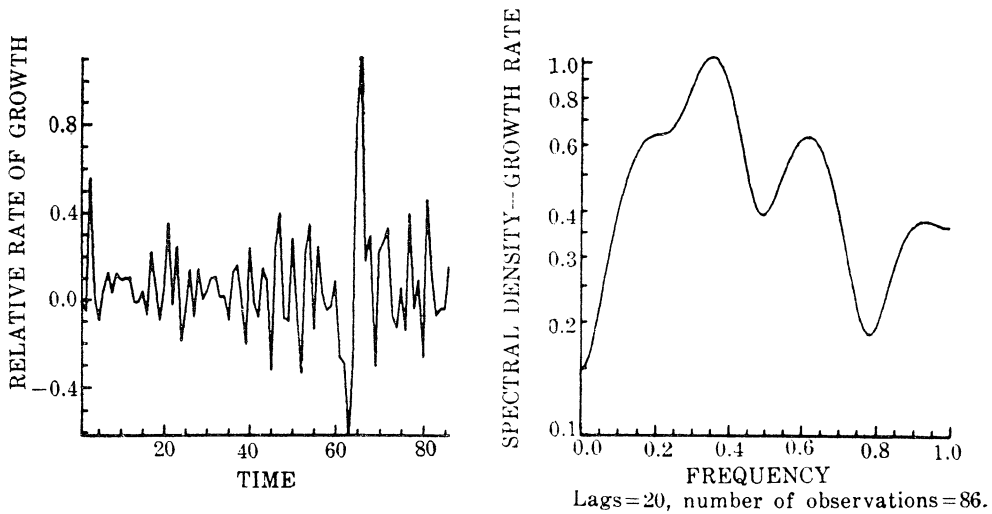


FIGURE 11

GROSS CAPITAL FORMATION, 1869-1955

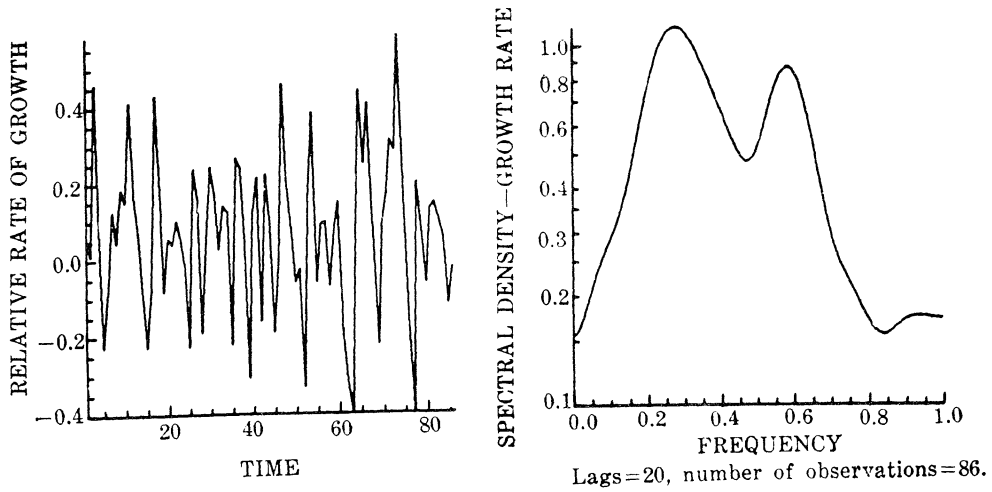


FIGURE 12  
GROSS PRODUCERS' DURABLES, 1869-1955

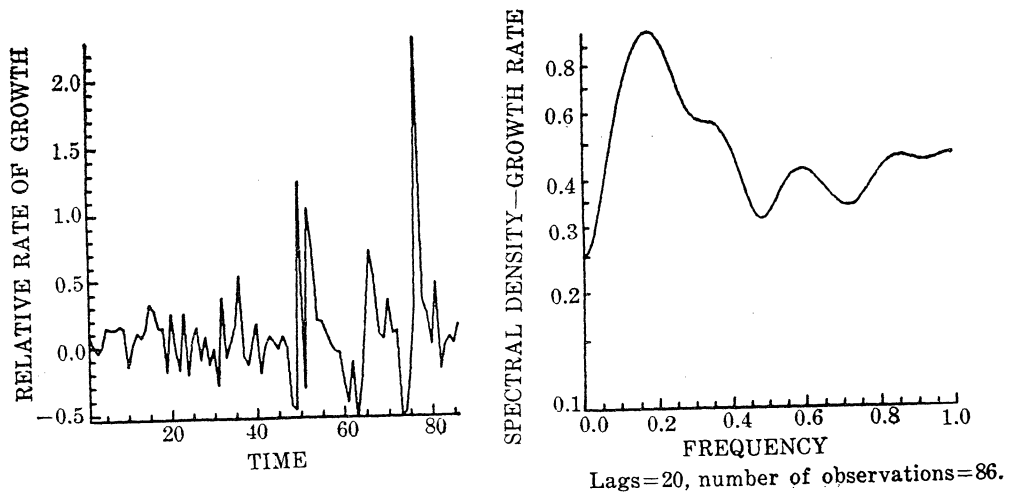


FIGURE 13  
GROSS NONFARM RESIDENTIAL CONSTRUCTION, 1869-1955

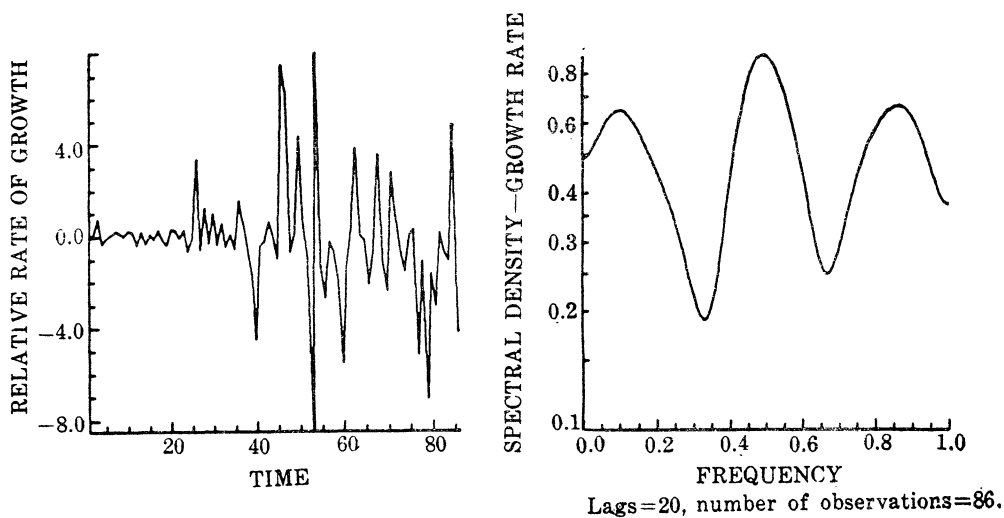


FIGURE 14  
INVENTORY INVESTMENT, 1869-1955

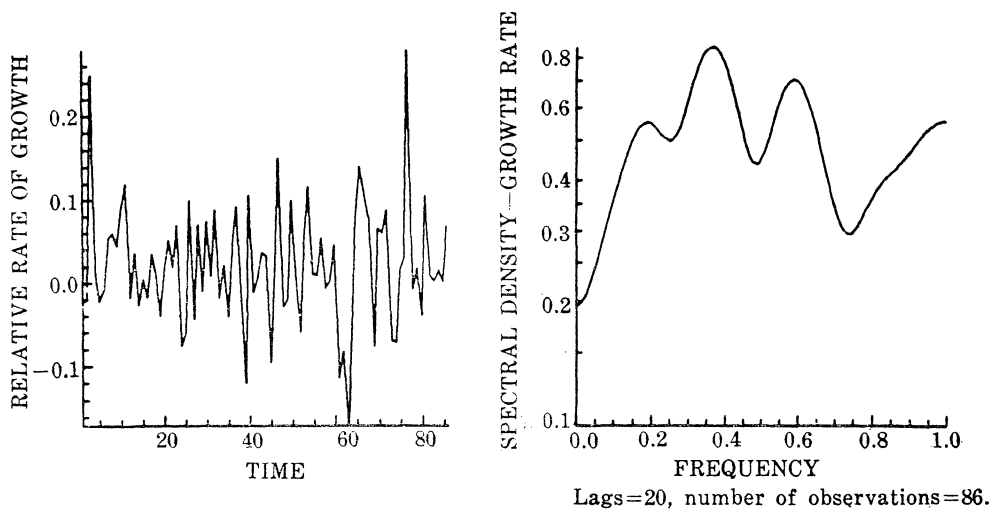


FIGURE 15  
GROSS NATIONAL PRODUCT PER WORKER, 1869-1955

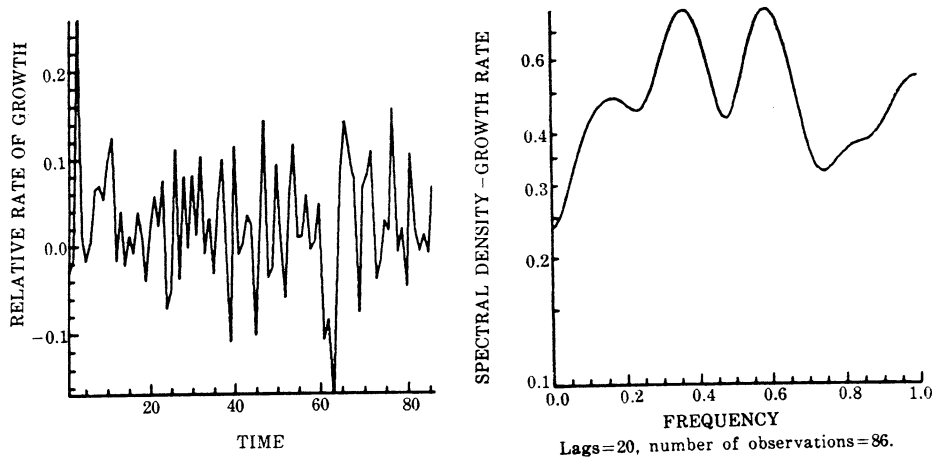


FIGURE 16  
GROSS NATIONAL PRODUCT PER CAPITA, 1869-1955

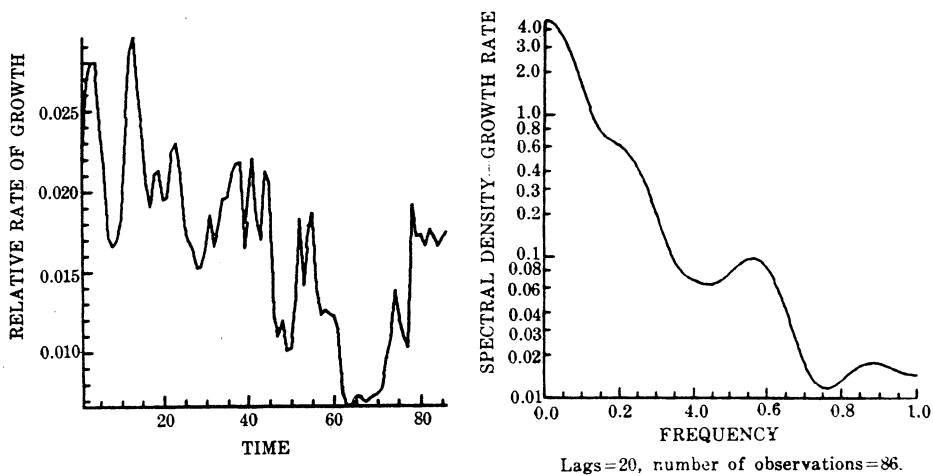


FIGURE 17  
TOTAL POPULATION, 1869-1955

long-swing peak in the total consumption and consumers' semi-durable series when there is no corresponding concentration of power in this neighborhood in the income series.

The spectral estimates of the investment series (8-11 in Table 3) shown in Figures 11-14 are interesting in several respects. The gross investment series, although similar to the production series (Figures 6-7) in terms of the location of the spectral peaks, is more strongly influenced by a [5.4-5.7] year fluctuation than are the production series. The gross producers' durables series exhibits important [7.1-8.0] and 3.4 ypc peaks and a very weak long-swing peak centered on 22.2 ypc. The gross nonfarm residential construction series exhibits the highly publicized long building cycle, although the periodicity of [11.8-12.5] ypc is somewhat shorter than that of previous estimates. It is very interesting to note that this frequency band contributes more than twice as much as any other to the variance of the series. The inventory investment series is interesting for two reasons. The periodicity of the business-cycle component emerges as 4.1-4.2 ypc, almost 0.5 ypc longer than in the other series. This is very curious in that a considerable amount of the explanation of business cycles, at least recently, has been centered around the inventory adjustment process.<sup>21</sup> The other interesting property of the inventory series is that, like the consumption series, it exhibits a long-swing peak.

The estimated spectra for GNP per worker and GNP per capita (Figures 15-16) closely resemble the spectrum of the GNP series. This is not particularly surprising in view of the fact that the spectrum of total population (Figure 17) closely resembles that of the GNP series except for the considerable amount of power concentrated near the zero frequency. As can be seen from the trace of the relative rate of growth of population, this concentration of power is probably due to the downward trend in mean. In order to explore this possibility, a linear trend was removed from the rate of growth of population. Even after detrending there remains a considerable concentration of power in the low-frequency end of the estimated spectrum.

#### 4. CONCLUSION

In this paper the growth-rate variant of the long-swing hypothesis has been explored. In order to determine the relative importance of long swings in the relative rate of growth, spectrum densities of a number of macroeconomic variables were estimated. These estimates, while not providing a definitive answer to the question of the existence of the Kuznets cycle, do nothing to dispel the skepticism which has been voiced in connection with the long-swing hypothesis. The spectrum peaks which do emerge in the long-swing frequency band are in most cases extremely weak; *in no case are they statistically significant.*

<sup>21</sup> This unexpected result may very well be explained by the inadequacy of the inventory investment series, especially during the earlier years of the series. Also, as mentioned in footnote 19, the estimated spectrum becomes flat when outliers are adjusted.



The estimated spectra do indicate that historically there have been relatively regular fluctuations in the rate of growth of aggregate activity which have been longer in duration than the ordinary business cycle. It does not seem to be possible to dismiss these fluctuations as purely random events in the sense that the spectrum peaks arise from sampling variability or from the existence of extreme values in the growth rate series. It was suggested that these longer fluctuations fit more conveniently into the major-cycle category than the long-swing category. These estimates, together with the observation that the usual filtering methods can "shift" a major-cycle peak into a long-swing peak in the power spectrum, tend to cast some doubt on the existence of long swings in the historical rate of growth of aggregate output in the U. S. economy.

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